Tutorial 9.

Preliminary:

Interest rate of return:

Net cashflows $C_0, C_1, \cdots, C_n$ at time $t_0, t_1, \cdots, t_n$, $t_0 = 0$

$$\sum_{k=0}^{n} C_k \cdot v^{t_k} = 0. \quad \text{because} \quad C_k = \underset{\uparrow \text{ cashin}}{A_k} - \underset{\uparrow \text{ cashout}}{B_k},$$

the present values are equal

$$A_0 + A_1 v^{t_1} + \cdots + A_n v^{t_n} = B_0 + B_1 v^{t_1} + \cdots + B_n v^{t_n} \Rightarrow \sum_{k=0}^{n} C_k \cdot v^{t_k} = 0.$$

$$\sum_{k=0}^{n} C_k v^{t_k} = C_0 + C_1(1+i)^{-t_1} + C_2(1+i)^{-t_2} + \cdots + C_n(1+i)^{-t_n} = 0$$

$$\Rightarrow (1+i)^{t_n}[C_0 + C_1(1+i)^{-t_1} + \cdots + C_n(1+i)^{-t_n}] = 0$$

$$C_0(1+i)^{t_n} + C_1(1+i)^{t_n - t_1} + \cdots + C_n(1+i)^0 = 0$$

$$\Rightarrow \sum_{k=0}^{n} C_k(1+i)^{t_n - t_k} = 0$$

5.1.1.

(a) $t_1 = 1$, $t_2 = 2$, $A_0 = 0$, $A_1 = 2.3$, $A_2 = 0$, $B_0 = 1$, $B_1 = 0$, $B_2 = 1.33$.

Set up the equation at t_2.

$C_0 = A_0 - B_0 = -1$, $C_1 = A_1 - B_1 = 2.3$, $C_2 = A_2 - B_2 = -1.33$.

$$C_0(1+i)^{t_2 - t_0} + C_1(1+i)^{t_2 - t_1} + C_2(1+i)^{t_2 - t_2} = 0$$

$$-(1+i)^2 + 2.3(1+i)^1 - 1.33 = 0.$$

Since $(2.3)^2 - 4(-1)(-1.33) = -0.03 < 0$, there is no solution for i.

(b) $t_1 = 1$, $t_2 = 2$, $A_0 = 0$, $A_1 = 2.3$, $A_2 = 0$. $B_0 = 1$, $B_1 = 0$. $B_2 = 1.32$

Set up the equation at time 0.

$C_0 = -1$, $C_1 = 2.3$, $C_2 = -1.32$. then

$-1 + 2.3v - 1.32v^2 = 0. \Rightarrow v = 0.91$ or 0.833. then

$i = 0.1$ or 0.2.

5.1.4.

Transaction A:

$$C_0^A v_A^0 + C_1^A v_A^1 + C_2^A v_A^2 + C_3^A v_A^3 = -5 + 3.72 v_A + 0 + 4 v_A^3 = 0$$

$$\Rightarrow v_A = 0.79789, \Rightarrow i_A = 0.25330$$

Transaction B:

$$C_0^B v_B^0 + C_1^B v_B^1 + C_2^B v_B^2 + C_3^B v_B^3 = -5 + 3 v_B + 1.7 v_B^2 + 3 v_B^3 = 0$$

$$\Rightarrow v_B = 0.79791, \Rightarrow i_B = 0.25328.$$

A is preferable when present value $P(C^A) > P(C^B)$, where $P(C^A) = \sum_{k=0}^{n} C_k^A v^{tk} = 0$

then $P(C_A) - P(C_B) = 0.72v - 1.7v^2 + v^3 > 0 \Rightarrow 0.72 - 1.7v + v^2 > 0$

$$\Rightarrow i < 0.1111, \text{ or } i > 0.25.$$

And B is preferable when $0.1111 \le i \le 0.25$.

5.1.3.

$A_0 = 0, \ B_0 = 5 \times 1000 = 5000, \Rightarrow C_0 = -5000,$

$A_1 = 0.2 \times 1000 = 200, \ B_1 = 200 \Rightarrow C_1 = 0.$ now we have $\frac{200}{4} + 1000 = 1050$ shares

$A_2 = 0, \ B_2 = 500 \times 4.5 = 2250 \Rightarrow C_2 = -2250,$ now we have $1050 + 500 = 1550$ shares.

$A_3 = 1550 \times (5 + 0.25) = 8137.5, \ B_3 = 0 \Rightarrow C_3 = 8137.5$

then $C_0 v^0 + C_1 v^1 + C_2 v^2 + C_3 v^3 = -5000 - 2250 v^2 + 8137.5 v^3 = 0$

$$\Rightarrow i = 0.049, \quad i^{(2)} = 0.098.$$

Additional Problem Set. 4.2.4, 4.2.5, 4.2.7   Tutorial: 5.1.1, 5.1.3, 5.1.4, 5.1.7

4.2.4.

yield rate $j = \frac{i^{(2)}}{2} = 3.3\%$, $BV_t = 90$, $F = 100$, $r = \frac{5\%}{2} = 2.5\%$.

then $BV_{t+1} = BV_t (1+j) - Fr = 90(1+3.3\%) - 100 \times 2.5\% = 90.47$

4.2.5.

The entry under Principal Repaid is called the amount for amortization of premium.
The amortization of a bond bought at a premium is also referred to as writing down a bond.

The premium $F(r-j)\,a_{\overline{n}|j} = 36$. $j = \frac{7\%}{2} = 3.5\%$

$PR = K - I$

$= F \cdot r - F \cdot j$

the present value $F(r-j)v_j^{n} = 1 \cdot v_j^{45} \Rightarrow F(r-j)v_j^{n} = 0.8714$

$\Rightarrow n = 26.113$ years.

4.2.7.

$PR_2 = 977.19$, $PR_4 = 1046.79$,

$PR_2(1+j)^2 = PR_4 \Rightarrow j = 0.035$ per half year.

$PR_1 = \frac{PR_2}{1+j} = 944.14$, then the premium is

$PR = PR_1\, s_{\overline{30}|0.035} = 48,739$